The semantics of modality

Day 2: Introduction to intensional semantics

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The reference of pronouns

Now take the following:

(1) He conquered Gaul.

The truth conditions of this sentence clearly depend on the reference of *he*.

(2) $[\![\text{He conquered Gaul}]\!] = 1 \leftrightarrow$ whoever *he* refers to conquered Gaul

Words such as *he* take their reference from the linguistic or extralinguistic context.
We'll deal with this by relativizing our denotation function to context:

(3) $[\![X]\!]^c =$ the denotation of X in context c.

For example, in a particular context:

(4) $[\![he]\!]^c =$ Caesar

Assignment functions

We'll be more precise about this. Since many things in a single utterance may depend on context, we'll assign a different index to each of these:

(5) He_1 thinks she_2 conquered Gaul.

Our *context* will be reduced to a contextual *assignment function*, g_c, which takes each index to its referent:

(6) $$g_c = \begin{cases} 1 & \rightarrow & \text{John} \\ 2 & \rightarrow & \text{Mary} \\ \ldots & & \end{cases}$$

So, given this particular definition of g_c:

(7) $$[\![\text{He}_1]\!]^{g_c} = \text{John}$$

Interpretation of bound pronouns

There are cases when these variable-reference elements are *bound* and their reference becomes fixed intra-sententially. Resumptive pronouns are one such case (but examples are complicated). PRO and traces are better examples:

(8) Caesar$_1$ wants PRO$_1$ to conquer Gaul.

(9) the Roman dictator$_1$ who t_1 conquered Gaul

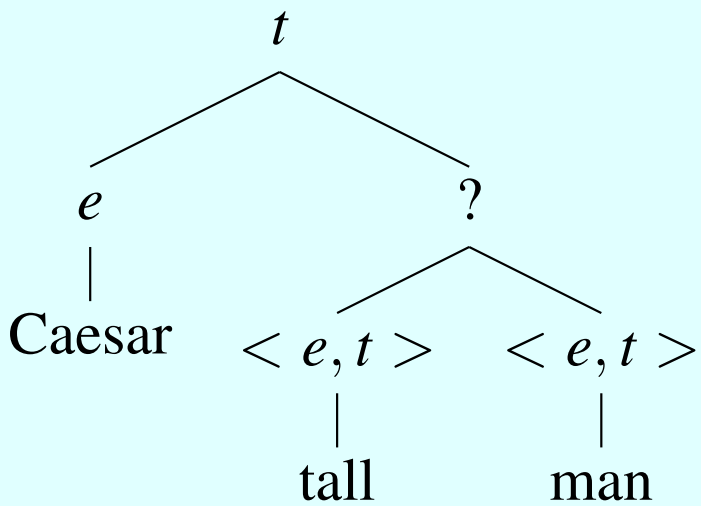
We'll fully work out the relative clause example. But first we need to understand adjectival modification and lambda abstraction.

Predicate modification

Take the following:

(10) Caesar was a tall man.

Informally, this has the meaning that *Caesar* is both *tall* and *a man*. Both *tall* and *a man* are predicates of type $< e, t >$. So how can we put this together?



(Yes, I'm assuming both *was* and *a* to be semantically vacuous.)

Predicate modification

There is no context for functional application.
We need a new rule:

(11) Predicate modification: if a node α has daughters β and γ both of type $< e, t >$, $[\![\alpha]\!]$ will be of type $< e, t >$, and have the meaning $[\![\beta]\!] \wedge [\![\gamma]\!]$.

The meaning is just the same as that of coordinated predicates.

Relative clause meaning

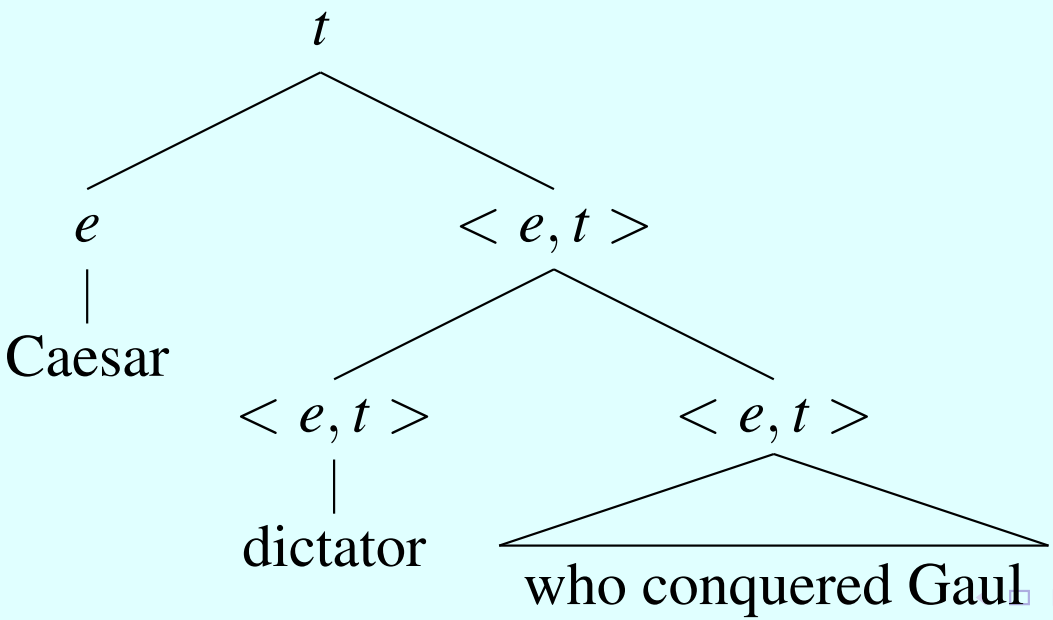
Take the following:

(12) Caesar was a dictator who conquered Gaul.

Like with adjectival modification, we'll assume that the meaning of this is:

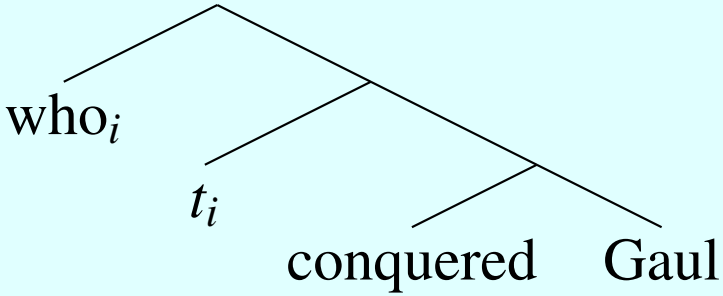
(13) $[\![$Caesar was a dictator who conquered Gaul$]\!] =$
$[\![$Caesar was a dictator$]\!] \wedge [\![$Caesar conquered Gaul$]\!]$

So:



Relative clause meaning

A relative clause creates a function out of any complete sentence.
Syntactically, in better-known languages this is usually done by movement of a relative pronoun:



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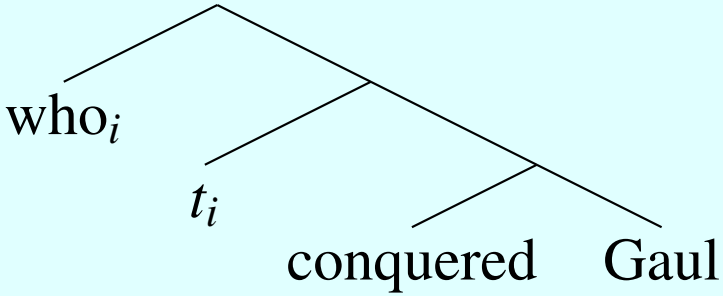

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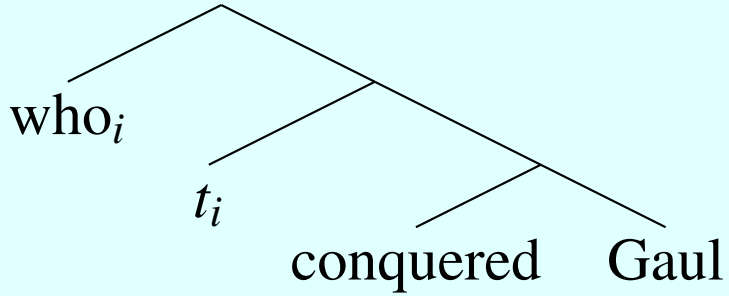

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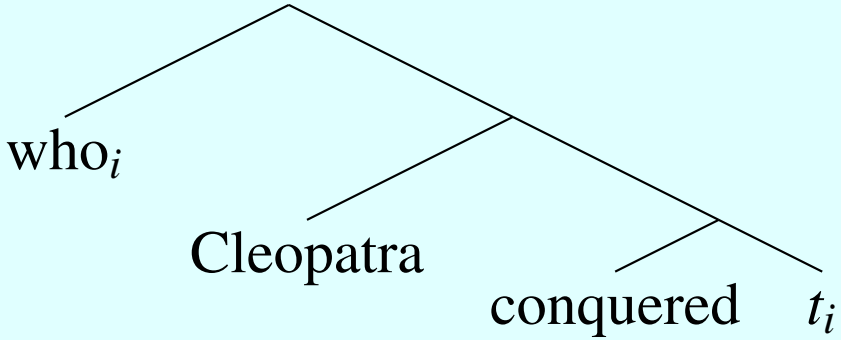
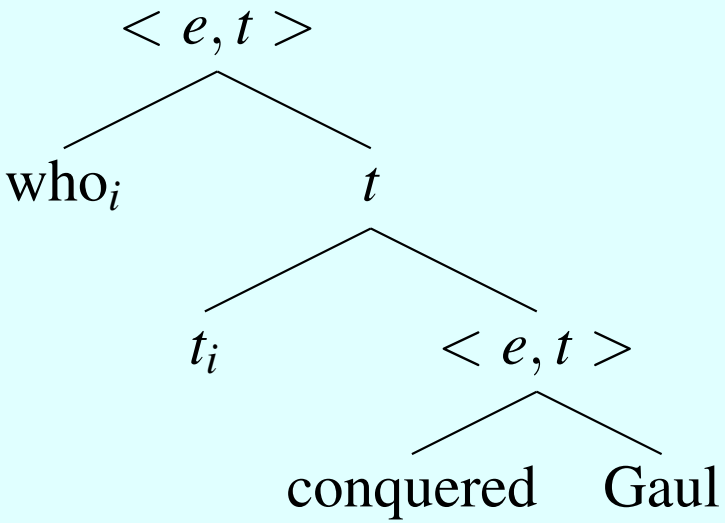


Relative clause meaning

So we want a way to turn a sentence (of type t) again into a predicate:



Lambda abstraction

We will deal with this by treating the trace as a pronoun, whose denotation is given by an assignment function:

(14) $[\![t_1]\!]^{g_c} = g_c(1)$

But how do we get this trace to act as a variable?

Lambda abstraction

We will deal with this by treating the trace as a pronoun, whose denotation is given by an assignment function:

(16) $[\![t_1]\!]^{g_c} = g_c(1)$

But how do we get this trace to act as a variable?
This is achieved by means of the *syncategorematic* rule of lambda abstraction.

(17) Lambda abstraction
Given two sister nodes $[\![\alpha]\!]^g$ and an index a, the denotation of the node dominating those two sisters is

$$\lambda x.[\![\alpha]\!]^{g/a \to x}$$

where $g/a \to x$ is the assignment function identical to g except for taking index a to x.

Contextualizing for time and place

Let's begin by considering the following sentence:

(18) It's raining.

In our system, this would have a meaning such as the following:

(19) $[\![\text{It's raining}]\!] = 1 \leftrightarrow$ it's raining

This is OK, but it treats the reference world for the assertion as unique and immutable. It is clear that the veracity of *It's raining* can only be ascertained with respect to a specific situation.

How do we formalize context?

We can try to deal with this with the same tool which we used to intepret a sentence such as the following:

(20) He is smart.

We claimed that the truth of this sentence can only be ascertained with respect to a particular context of utterance. We represented the particular aspect of context involved in the interpretation of pronouns with an assignment function, g_c.

(21) The partial function $g_c(x)$ takes an index to its referent.

(22) $[\![\text{He}_1 \text{ is smart}]\!]^{g_c} = \text{smart}(1) = \text{smart}(\text{John})$ under the assignment $g_c(1) = \text{John}$

What else is the context?

Let's extend this idea to formalize other aspects of context.
It's not only pronouns that may be deictic or anaphoric. Other words have these properties:

(23) Locative expressions: *here* (deictic), *there* (deictic and anaphoric), ...

(24) Temporal expressions: *now* (deictic), *then* (normally anaphoric), ...

In fact, more abstract things can be deictic or anaphoric in this way:

(25) Deictic tense: *it's raining*.

(26) Anaphoric tense: *when I arrived, she was leaving*.

So our context of utterance should include at least time and spatial location.
We will not distinguish in our formalism between context established deictically and context established anaphorically. The important distinction will be, as with pronouns, between bound and free elements.

What else is in the context?

So let's examine something like this:

(27) He_1 is going now.

We propose that there are two elements to which the denotation is relativized.
The assignment function g_c which we saw above, and the utterance time, t_0.
We want the meaning of the preceding sentence to be something like this:

(28) $[\![\text{He}_1 \text{ is going now}]\!]^{g_c,t_0} = g_c(1) \text{ is going at } t_0$

I won't work the compositional semantics of this today.
We could do the same with expressions having to do with spatial location.

What else is in the context?

OK, but how about the following?

(29) He went home.

The event is clearly in the past, not in the utterance time. The time in the past might be unknown and irrelevant, but it may also be set by utterance context:

(30) After work I invited him for dinner, but he was tired. He went home.

In this case, the time is *some time just after work*. It could be more or less precise than this, but it's usually more precise than just "any time in the past", as the lack of specificity of English tense morphology might lead us to believe. This time is what's called the *reference* or *topic* time.

What else is in the context?

So at least two times are part of context: the time that we are talking about, t^*, and the time at which we are talking, t_o. In the case of utterances in the present tense, $t^* = t_o$, while in utterances in the past tense $t^* < t_o$.
In fact, there exist two temporal adverbs each of which picks one of these two times. Informally:

(31) $[\![\text{now}]\!]^{t_o,t^*} = \text{at } t_o$

(32) $[\![\text{then}]\!]^{t_o,t^*} = \text{at } t^*$

We'll talk a bit more about tense tomorrow, when we introduce modality in the imperfective.

What else is in the context?

Do we need to put anything else in the context? von Fintel and Heim give a suggestive example:

(33) A detective lives at 221B Baker Street.

Somehow, we would like to be able to relativize the truth of this sentence to the imaginary world of Sherlock Holmes. In such a world, a number of statements are true which are false in the actual world.
So we might want to include another type of variable determined by context, one ranging through possible worlds or possible situations.

Evaluation with respect to worlds

So:

(34) $[\![\text{A detective lives at 221B Baker Street}]\!]^w = 1 \leftrightarrow$
a detective lives at 221B Baker Street in w

This is not a trivial change. The fact that we plunge into the novel allows us to evaluate the truth of this statement with respect to a world that is not our world.

What really matters here

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When we know that statements pertain to a different world because they are contained in the novel that we are holding, we are, as it were, freely setting the world or situation variable to something that is relevant. However, in a sentence such as the following, the choice of worlds or situations is highly constrained:

An example

(35) John believes that the earth is flat.

This statement might be true in the actual world, regardless of the fact that the statement *the earth is flat* is clearly false in the actual world.
The sentence is true if the statement *the earth is flat* is true in the world(s) compatible with John's beliefs.

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This is a situation in which the contextual variable for worlds or situations is bound intra-sententially:

(39) $[\![\text{the earth is flat}]\!]^w =$ the earth is flat in w

(40) In the context of *John believes that* . . . , w is John's belief world.

Abstracting to bind a pronominal trace

Remember that it is possible to abstract over the assignment to deal with cases where the pronoun is bound, hence assignment-independent. Informally:

(41) $[\![\text{Caesar}_1 \text{ is who}_1 \text{ Cleopatra conquered } t_1]\!] =$
$[\lambda x.x \text{ is such that Cleopatra conquered } x](Caesar)$

This was achieved by means of the syncategorematic rule of lambda abstraction.

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Given two sister nodes $[\![\alpha]\!]^g$ and an index a, the denotation of the node dominating those two sisters is

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Abstracting to bind a world variable

We can explore something similar for a sentence such as:

(43) John thinks the earth is flat.

If the embedded clause were an independent predicate, we could say that its semantics is:

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In this case, w_o represents the actual world.
However, this reference world is manipulated in sentences such as (43):

(48) $[\![\text{the earth is flat}]\!]^{w} = 1 \leftrightarrow$ the earth is flat in w

That is, the truth is relativized to the reference world w rather than to the actual world w_o. Just as we did with assignment functions, we'll bind this w variable to get it to equal John's belief world.

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Type	Typical variable names	Typical constant names
e	x, y, z	a, b, c
t	...	...
s	w, w'	w^{o}, w^{*}
functions	P, Q, R	...

Formalizing this

Now let's apply the rule of lambda abstraction over the world variable:

(49) $\lambda w.[\![\text{the earth is flat}]\!]^w = \lambda w.\text{the earth is flat at } w$

This has the form of a predicate of worlds. Its semantic type is $< s, t >$. Presumably, it could combine with a world, just like a predicate of entities combines with a subject to say something about it.

Formalizing this

When does the rule of lambda abstraction apply?
With pronouns, we applied the rule of abstraction whenever an index was present in the structure. The presence of this index is loosely associated with NP movement.
With the world variable, lambda abstraction occurs in what are called *intensional contexts*. I will define intensional contexts extensionally:

Complements of propositional attitude verbs: *believe*, *think*, *doubt*, ...

Complements of modal auxiliaries: *should*, *may*, *could*, ...

There are other intensional contexts, but these will do for now.

Formalizing this

Let's compare a propositional attitude verb with a regular transitive verb:

(50) I saw John.

(51) I believe the world is flat.

The types of these verbs have to differ. The first is, as we said previously, of type $< e, < e, t >>$. The second takes a clause as a complement. But is this clause of type t?

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We might not have any grounds to object to it being of type t with verbs such as *know* (in fact, as we discussed in class, this is clearly more complicated):

(55) John knows that the earth is not round.

In this case, the truth of *the earth is not round* is computed with respect to the actual world, not with respect to an imaginary world. Other *factive* verbs include *acknowledge*, *accept*, etc.